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CONFIDENCE INTERVALS FOR THE RELIABILITY OF A FUTURE SYSTEM CON--ETC(U)
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An inferential procedure is presented which provides confidence intervals for a future reliability parameter when reliability growth testing is only partially completed. Hypothesis tests based on this method are uniformly most powerful unbiased. These results are applicable if (1) the system failure rate can be modeled as the intensity function of a Weibull process and (2) efforts to improve reliability are assumed to continue at a steady rate throughout the intervening period of testing. The usefulness of this methodology is

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illustrated by evaluating the risk of not reaching some future reliability milestone. If such risk is unacceptably high, program management may have time to identify problem areas and take corrective action before testing has ended. As a consequence, a more reliable system may be developed without incurring overruns in the scheduling or cost of the development program.

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CONFIDENCE INTERVALS FOR THE RELIABILITY OF A FUTURE SYSTEM CONFIGURATION

## 1. INTRODUCTION

Reliability growth management is a critical function in the development programs of major defense systems.[1] It consists of planning, monitoring, and controlling the growth of reliability parameters throughout system development in order to achieve the reliability milestones for each test phase and for the overall program. A key factor in this process is the ability to assess the risk of not meeting a reliability requirement and to make such assessment at an early stage in the current test phase. If this risk is unacceptably high, the program manager may then have an opportunity to take remedial action before test time or other program resources are exhausted. The risks of failing to achieve program goals or contractual requirements can therefore be minimized. Instead of having to react to program shortcomings after the fact, management can exert positive control over the growth process to accomplish reliability objectives.

Reference 1 (pp. 10, 23, 28, 64-66, 75-78) discusses the use of reliability growth models to project reliability estimates beyond the present test time to some future time, such as the end of the current test phase. These projections are valid only if test conditions remain relatively constant and the development effort continues at its previous level. The projected reliability estimates are compared with future milestones in order to assess whether the reliability enhancement program is likely to reach a successful conclusion.

One of the problems with assessing a program by this method is how to evaluate the accuracy of the reliability projections. Such projections are only point estimates and do not reflect the uncertainties that accompany random sampling from a probabilistic model. In this paper we show how to quantify these uncertainties when the Weibull process is used to model and forecast reliability growth. The result is an objective appraisal of current program risks, and this appraisal can be factored into those management decisions which may impact on future reliability parameters.

The Weibull process model has been successfully applied to the reliability test results of many complex defense systems. It is introduced in Section 2 in a parametric form that is especially suited to the problem of forecasting. The basic features of this model are described in Appendix C of Reference 1, which includes confidence interval procedures for the reliability of the current system configuration. (See also References 2 and 3.) The theory developed in Section 3 extends these latter results to provide inferential

[1]Department of Defense, Reliability Growth Management, Military Handbook 189, Naval Publications and Forms Center, Philadelphia, PA, February 1981.

[2]Bain, L. J. and M. Engelhardt, "Inferences on the Parameters and Current System Reliability for a Time Truncated Weibull Process," Technometrics, Vol. 22, pp. 421-426, August 1980.

[3]Crow, L. H., Confidence Interval Procedures for Reliability Growth Analysis, Technical Report No. 197, U S Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, MD, June 1977.

procedures for future reliability levels. These procedures are illustrated in Section 4, where confidence intervals are obtained for the reliabili[illegible] to be achieved at future points in a test phase which is still in progress. [illegible] obtained by an equivalent technique is the risk of not achieving a certain reliability level at the end of the test phase.

## 2. SPECIFICATION OF MODEL

Consider a reliability growth test phase which has been underway for T units of testing. We shall hereinafter regard these test units as time, although they could equally well represent other units such as distance. Suppose that the test phase began at time 0, but is planned to continue for an additional S units of testing till test time T+S, at which point the system configuration will have failure rate R. Our objective is to make inferences about the parameter R.

A Weibull process is a nonhomogeneous Poisson process with an intensity function that can be expressed as a multiple of some power of the test time. For the particular test phase described above, an intensity function of the appropriate parametric form is

$$r(t) = R[t/(T+S)]^{\beta-1}, \tag{1}$$

where $R>0$, $\beta>0$, and $0<t\leq T+S$. As shown in Figure 1, the function $r(t)$ models the failure rate of the system configuration as it changes over a reliability growth test phase of length T+S, and the failure rate at the end of the (as yet uncompleted) test phase is given by $r(T+S) = R$.

The failure rate model in Figure 1 shows a decreasing trend during future testing from time T to time T+S. This trend reflects our previously stated intention to continue reliability improvements throughout this period. The case in which reliability is constant from T to T+S is treated in Reference 4.

According to the scenario of this paper, test results are available for the test period from time 0 to the (current) time T, but the system testing from time T to time T+S has not yet been accomplished. Let N be the number of failures that occur before time T and $T_1, \ldots, T_N$ the observed failure times $(0<T_1 < \ldots <T_N<T)$. Then the Poisson process with intensity function $r(t)$ has a sample function density given by

$$f_{N,T_1,\ldots,T_N}(n, t_1, \ldots, t_n)$$

[4]Miller, G., "Efficient Methods for Assessing Reliability," Proceedings of the Nineteenth Annual U S Army Operations Research Symposium, Part III, pp. 33-42, October 1980.

$$= \begin{cases} \exp[-(RT/\beta)Q^{1-\beta}] & \text{if } N = 0, \\ R^n \prod_{i=1}^{n} [(T+S)/t_i]^{1-\beta} \exp[-(RT/\beta)Q^{1-\beta}] & \text{if } N = n>0, \end{cases} \quad (2.1)$$

where $Q = (T+S)/T$; $n = 0, 1, \ldots$; and $0<t_1 < \ldots < t_n<T$. (See e.g., Reference 5.)

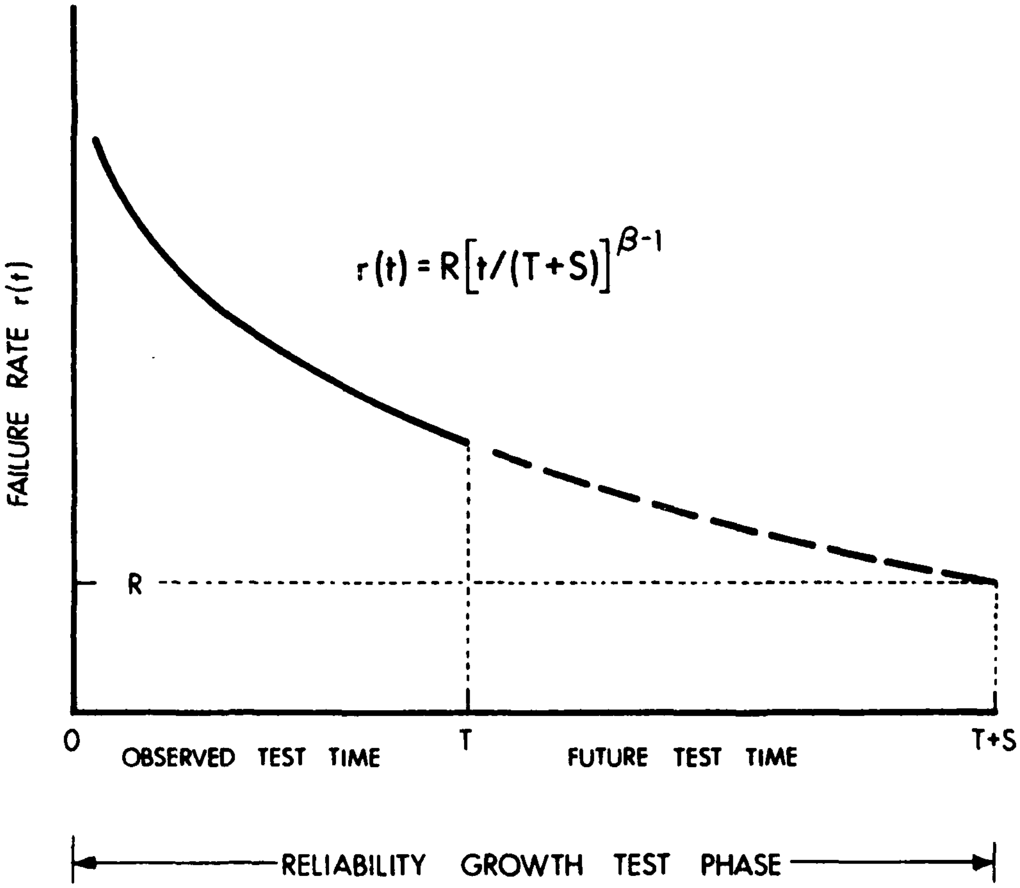


FIGURE 1. Intensity Function for the Case $\beta<1$.

[5]Snyder, D. L., Random Point Processes, John Wiley and Sons, New York, NY, 1975.

## 3. DERIVATION OF RESULTS

### 3.1 Point Estimators

The Weibull process model is used in applications where Pr(N = 0) is quite small, and therefore the likelihood expression in Equation (2.2) can be maximized to obtain point estimators for $\beta$ and R as follows:

$$\hat{\beta} = N / \sum_{i=1}^{N} \ln(T/T_i), \tag{3}$$

$$\hat{R} = N\hat{\beta}Q^{\hat{\beta}-1}/T. \tag{4}$$

As would be expected, the expression in (3) is identical to the estimator for $\beta$ in Reference 2 (Equation (4)). The projected mean time between failures (MTBF) for the system configuration at the end of the test phase (time T+S) is estimated by $\hat{R}^{-1}$.

The point estimators in Equations (3) and (4) are convenient because of their simplicity, but were obtained without conditioning formally on the event N>0. As a practical matter, inferences on the two-parameter Weibull process are possible only when N>0, and we shall condition on this event in the sequel without further mention.

### 3.2 Reduction of the Parameter Space

Let $V = \sum_{i=1}^{N} \ln[(T+S)/T_i]$, and observe from Equation (2) that V is a sufficient statistic for $\beta$. It follows from Reference 6 (pp. 134-140) that uniformly most powerful unbiased (UMPU) hypothesis tests on the future failure rate R can be constructed by utilizing the conditional distribution of N given V=v. To obtain this distribution, we begin by determining the conditional distribution of V given N=n.

Given N=n, the random variables $T_1, \ldots, T_n$ are distributed as the order statistics from n independent distributions with cumulative distribution function

$$F(t) = \int_0^t r(x)dx / \int_0^T r(x)dx$$

$$= (t/T)^{\beta}, \tag{5}$$

[6] Lehmann, E. L., Testing Statistical Hypotheses, John Wiley and Sons, New York, NY, 1959.

where $0<t<T$. Let $X$ be a random variable with distribution function $F$. Then a straightforward calculation shows that the random variable $\ln[(T+S)/X]$ is distributed over the interval $(\ln Q,\infty)$ according to

$$\Pr\{\ln[(T+S)/X] \le y\} = 1-\exp[-(y-\ln Q)\beta], \qquad (6)$$

where $\ln Q<y<\infty$. This latter function is a two-parameter exponential distribution function on the interval $(\ln Q,\infty)$. The conditional distribution of $V$ given $N=n$ is therefore the sum of $n$ such distributions, all independent, and consequently is a three-parameter gamma distribution with density function

$$f_{V|N}(v|n)$$

$$= \beta^n(v-n\ln Q)^{n-1}\exp[-\beta(v-n\ln Q)]/(n-1)!, \qquad (7)$$

where $n\ln Q<v<\infty$.

The random variable $N$ is Poisson distributed with mean value $\theta\equiv(RT/\beta)Q^{1-\beta}$, so that (conditional on $N>0$)

$$\Pr(N=n) = [1-\exp(-\theta)]^{-1}\theta^n\exp(-\theta)/n!, \qquad (8)$$

$n = 1, 2, \ldots$ . Thus the joint density function of $V$ and $N$ is

$$f_{V,N}(v,n) = f_{V|N}(v|n)\ \Pr(N=n)$$

$$= \frac{\exp(-\theta-\beta v)}{1-\exp(-\theta)}\ (RTQ)^n\ \frac{(v-n\ln Q)^{n-1}}{n!(n-1)!}, \qquad (9)$$

where $n = 1, 2, \ldots$ and $n\ln Q<v<\infty$.

In the case $S=0$ (forecasting zero time into the future), we see that $\ln Q=0$ and that the results in this paper generalize certain results in [3] and [2] on inferences for current system reliability. In the case $S>0$, the above inequality $n\ln Q<v<\infty$ implies that $N$ has finite support, given $V=v$:

$$\Pr(0<N<v/\ln Q|V=v) = 1. \qquad (10)$$

Given $V=v$, let $G(v,S)$ be the greatest integer less than $v/\ln Q$ if $S>0$ and $G(v,S) = \infty$ if $S = 0$.

We can now write down the conditional distribution of $N$ given $V=v$ as

$$p(n;R) \equiv \Pr(N=n|V=v,\ N>0)$$

$$= \frac{(RTQ)^n(v-n\ln Q)^{n-1}/n!(n-1)!}{\sum_{k=1}^{G(v,S)} (RTQ)^k\ (v-k\ln Q)^{k-1}/k!(k-1)!}\ , \qquad (11)$$

where $n = 1, 2, \ldots, G(v,S)$. This expression for $p(n;R)$ can be readily [illegible] l-uated at minimal cost with an electronic computer.

### 3.3 Inferential Procedures

A conservative $1-\alpha$ confidence interval for $R$ can be constructed by obtaining values $R_1$ and $R_2$ which satisfy $\sum_{k=n}^{G(v,S)} p(k;R_1) = \alpha_1$ and $\sum_{k=1}^{n} p(k;R_2) = \alpha_2$, where $\alpha_1 + \alpha_2 = \alpha$. The corresponding confidence bounds for $R^{-1}$ (the MTBF at test time T+S) are $R_2^{-1}$ and $R_1^{-1}$. Because N is a discrete random variable, construction of exact confidence intervals would require randomization. A UMPU test of $H_0{:}R \le R_0$ versus $H_1{:}R > R_0$ at significance level $\alpha$ calls for rejection of $H_0$ if $\sum_{k=n}^{G(v,S)} p(k;R_0) \le \alpha$. Other UMPU hypothesis tests can be constructed in a similar manner. If $R_0^{-1}$ is the MTBF goal for the end of the test phase (time T+S), then the risk of not achieving this goal may be evaluated as $\sum_{k=1}^{n} p(k;\, R_0)$.

## 4. EXAMPLE

Suppose that a reliability growth test phase has been in progress for T=200 hours and is scheduled to continue for another S=200 hours. From the test data up to time 200, we wish to obtain an 80 percent confidence interval for the MTBF at time T+S = 400. The following failure times $t_i$ were recorded (n=21): 2.2, 3.3, 4.5, 5.3, 5.8, 20.3, 27.4, 34.1, 55.2, 58.4, 61.4, 62.[illegible], 78.3, 78.4, 91.9, 97.7, 112.4, 116.9, 142.4, 176.8, 181.5.

Equations (3) and (4) yield $\hat{\beta} = .591$ and $\hat{R}^{-1} = 21.4$, and it is also of interest to observe that $v/\ln Q = 72.3$. Thus $G(v,S) = 72$, so that the set of positive integers less than or equal to 72 is a support of the conditional distribution of N given V=v.

With Equation (11) we obtain by iteration the values $R_2^{-1} = 12.7$ and $R_1^{-1} = 38.6$ such that $\sum_{k=21}^{72} p(k;R_1) = .10$ and $\sum_{k=1}^{21} p(k;R_2) = .10$. The interval (12.7, 38.6) is therefore an 80 percent confidence interval for the MTBF at time 400.

By successively taking S=0 and S=100, we can obtain in a similar manner 80 percent confidence intervals (10.7, 26.0) and (11.9, 32.7) for the MTBF at times 200 and 300, respectively. All three confidence intervals are shown in Figure 2 for comparison purposes.

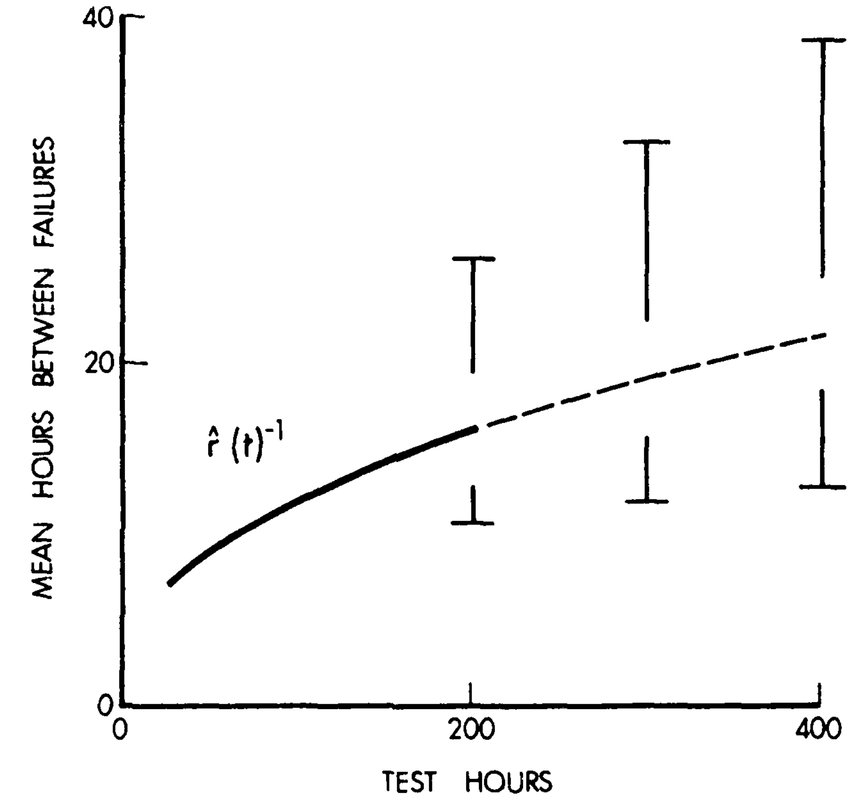


FIGURE 2. Eighty Percent Confidence Intervals for Current and Future MTBF Parameters.

Suppose further that an MTBF goal of 15.0 has been set as a milestone for the end of the current reliability growth test phase (T+S = 400). Based on the data up to time T = 200, the risk of not achieving this goal is $\sum_{k=1}^{21} p(k;15^{-1}) = .20$. In view of such a result, the program manager should feel optimistic about this aspect of the development program, but will probably want to avoid any actions which might adversely affect the overall reliability enhancement effort.

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